A GKM Description for the Equivariant Coinvariant Ring of a Pseudo-Reflection Group

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Let $\mathbb{F}$ be a field, $V = \mathbb{F}^n$ a finite dimensional vector space, $R = \mathrm{Sym}(V^*)$ the ring of polynomial function on V, and let $W \subset \mathrm{GL}(V)$ be any finite subgroup of invertible linear transformations on V. If $R^W \subset R$ denotes the subring of invariant polynomials, we define the equivariant coinvariant ring of W to be the tensor product $R \otimes_{R^W} R$. There is a well defined map $\mu\colon R \otimes_{R^W} R \to \mathrm{Maps}(W, R)$ defined by $\mu(f \otimes g)(w) = (f \cdot w(g))$ which we call the decomposition map of W. We would like to understand for which groups is μ injective, and for those groups, what is the image? Motivation for this problem stems from topology, and GKM theory offers some answers. Specifically, if W is a Weyl group associated to some flag manifold $X = G/T$, then, with respect to its natural torus action, the equivariant cohomology ring $H_T(X)$ can be identified with the equivariant coinvariant ring $R \otimes_{R^W} R$ where $V = \mathfrak{t}$ the Lie algebra of T. Moreover, the decomposition map can be identified with the map on equivariant cohomology rings induced by inclusion of the fixed point set $X^T = W \subset X$, and a theorem of Borel implies that map is injective. Finally a theorem of Goresky-Kottwitz-MacPherson then identifies the image of the decomposition map as "compatible polynomial maps" on the GKM 1-skeleton of W, i.e.

$$R \otimes_{R^W} R \cong \mathrm{im}(\mu) = \left\{ F \in \mathrm{Maps}(W, R) \mid F(w) - F(w s_\gamma) \equiv 0 \bmod w(\gamma)\ \forall\ s_\gamma \in \mathrm{ref}(W) \right\}.$$

In this talk, I will discuss an extension of these results to the family of groups called (non-modular) pseudo-reflection groups. Along the way I will discuss the rudimentary facts underlying GKM theory, and give several examples. Some the results presented here are based on joint work with L. Smith at Georg-August-Universität Göttingen, Germany and J. Watanabe at Tokai University, Japan.