

## Algebraic Topology – Homework 8

Due date : May 29th in class.

### **Exercise 30. (16 Points)**

Let  $X$  be a topological space endowed with the structure of a CW complex. Prove that  $H_n(X^n)$  is a free abelian group for every  $n$ . (Here  $X^n$  denotes the  $n$ -skeleton of the CW complex.)

### **Exercise 31. (8+10 Points)**

Let  $X^1$  be the wedge of two circles  $S^1 \vee S^1$ , where one circle is labeled by  $a$ , and one by  $b$ .

- (i) Let  $X$  be the two-dimensional CW complex obtained by attaching a 2-cell to  $X^1$ , where the attaching map is  $ab^2a^2b^2a^{-1}b^{-1}$ . Compute the (cellular) homology of  $X$ .
- (ii) Let  $Y$  be the two dimensional CW complex obtained by attaching two 2-cells to  $X^1$ , with attaching maps respectively given by  $a^5b^{-3}$  and  $b^3(ab)^{-2}$ . Compute the (cellular) homology of  $Y$ .

### **Exercise 32. (16 Points)**

For  $SX$  the suspension of  $X$ , show by a Mayer-Vietoris sequence that there are isomorphisms  $\tilde{H}_n(SX) \cong \tilde{H}_{n-1}(X)$  for all  $n$ .