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Symplectic Geometry – Homework 8

Due on June 15th 2015, in class

Exercise 1.

Determine all possible affine connections on $\mathbb{R}^n$. Of these, determine those whose geodesics are straight lines $c(t) = at + b$, where $a, b \in \mathbb{R}^n$.

Exercise 2.

Let H be the upper half plane in $\mathbb{C}$, i.e.

$$H = \{z = x + iy \in \mathbb{C} \mid y > 0)\} .$$

Define the metric

$$g = \frac{1}{y^2} (dx \otimes dx + dy \otimes dy) .$$

- Compute the Christoffel symbols of this metric.
- Give the geodesic equations
- Show that the curves $\alpha(t) = (0, e^t)$ and $\beta(t) = (\tanh(t), \frac{1}{\cosh(t)})$ are geodesics. What are the sets $\alpha(\mathbb{R})$ and $\beta(\mathbb{R})$?

For $M \in GL(2, \mathbb{R})$ we write

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The group $GL(2, \mathbb{R})$ acts on H by Möbius transformations as follows

$$M \cdot z = \frac{az + b}{cz + d}$$

- Show that for each $M \in GL(2, \mathbb{R})$ the map $z \longmapsto M \cdot z$ is an isometry.
- Using the fact that isometries map geodesics to geodesics, try to draw the geodesics of H.

Remark This is the famous hyperbolic plane. This is isometric to the Poincaré disc, which you have surely seen pictures of. For example Escher's Circle limit III depicts this.