Mathematisches Institut | Universität zu Köln
Prof. Dr. Sabatini | November 12th 2014

Algebraic Topology – Homework 5

Due date : November 19th in class

Exercise 1.

(1) Recall that the **wedge sum** $X \vee Y$ of two topological spaces X and Y, with given points $x \in X$ and $y \in Y$, is the quotient of the disjoint union $X \amalg Y$ obtained by identifying x with y. (Here you *cannot* use Proposition 1 in Exercise 4, you need to do it directly.)

Endow the wedge of g circles with the "obvious" Δ-complex structure (which has exactly g 1-simplices) and compute its simplicial homology.

(2) Endow S^2 with a Δ-complex structure with two Δ^2 simplices identified along their boundaries with identity map. Compute the simplicial homology of S^2.

Exercise 2.

Exercise number 5 page 131 on Hatcher's book. (Click here for Chapter 2.)

Exercise 3.

Let X_n be the topological space obtained from an n-gon with identifications on the boundary induced by the word $\overbrace{a \cdot a \cdot \cdots \cdot a}^{n \text{ times}}$. (For example, X_2 is $\mathbb{RP}^2$.) This space is also called **n-fold dunce cap**.

(a) Compute $\pi_1(X_n)$.

(b) Endow X_n with a suitable Δ-complex structure, and compute its simplicial homology groups. (Hint : Consider the regular polygon with n edges. Consider the Δ-complex structure obtained by adding a vertex in the middle of it, and 1-simplices pointing radially inward.)

Exercise 4.

In this exercise you will need the following

Proposition 1. *Let X_α be topological spaces endowed with a Δ-complex structure, and consider $\vee_\alpha X_\alpha$, which can also be endowed with a Δ-complex structure. Assume that each of the point $x_\alpha \in X_\alpha$, identified in the wedge sum $\vee_\alpha X_\alpha$, has a contractible neighborhood in X_α. Then $H_i^\Delta(\vee_\alpha X_\alpha) \cong \bigoplus_\alpha H_i^\Delta(X_\alpha)$ for every $i > 0$.*

Given finitely generated abelian groups G_1 and G_2, with G_2 free, describe a finite 2-dimensional Δ-complex X which is connected and such that $H_1^\Delta(X) \cong G_1$ and $H_2^\Delta(X) \cong G_2$. (Hint : use the *fundamental theorem of finitely generated abelian groups*.)