Mathematisches Institut
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Universität zu Köln
November 26th 2014

Algebraic Topology – Homework 7

Due date : December 3rd in class

<u>Exercise</u> 1.

Complete (without looking at the book !) the proof of the 'Snake Lemma' done in class, namely prove that $\operatorname{Im}\delta = \operatorname{Ker} i_*$.

<u>Exercise</u> 2.

— Exercise number 15 on page 132 of Hatcher's book.
— For exercise 17 (b) on page 132 of Hatcher's book, compute $H_i(X, A)$ and $H_i(X, B)$ only for $i = 0, 1$.
— Assume that $H_n(S^1) = 0$ for all $n \geq 2$. Compute the relative homology groups $H_n(X, \partial X)$ for all $n \geq 0$, where $X = [0, 1] \times S^1$ and $\partial X = (\{0\} \times S^1) \cup (\{1\} \times S^1)$

<u>Exercise</u> 3.

In this exercise you will prove the so called '**Five Lemma**'. Suppose that the following diagram of abelian groups commutes :

$$\begin{array}{ccccccccc} A & \xrightarrow{\alpha} & B & \xrightarrow{\beta} & C & \xrightarrow{\gamma} & D & \xrightarrow{\delta} & E \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle g} & & \downarrow{\scriptstyle h} & & \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle j} \\ A' & \xrightarrow{\alpha'} & B' & \xrightarrow{\beta'} & C' & \xrightarrow{\gamma'} & D' & \xrightarrow{\delta'} & E' \end{array}$$

The Five Lemma asserts that if the rows are exact, and f, g, i, j are isomorphisms of groups, then h is as well.

(i) Prove the Five Lemma (without looking at the book !)
(ii) What are the *minimal* conditions needed on f, g, i, j that ensure h to be surjective ?
(iii) What are the *minimal* conditions needed on f, g, i, j that ensure h to be injective ?