Mathematisches Institut Universität zu Köln
Prof. Dr. Sabatini January 14th 2015

Algebraic Topology – Homework 12

Due date : January 21st in class

Exercise 1.

Let A, B and C be abelian groups, and $A \xrightarrow{i} B \xrightarrow{j} C \to 0$ an exact sequence. Prove that the 'dual sequence'

$$\mathrm{Hom}(A; G) \xleftarrow{i^*} \mathrm{Hom}(B; G) \xleftarrow{j^*} \mathrm{Hom}(C; G) \leftarrow 0$$

is exact.

Exercise 2.

Let Σ be a compact surface endowed with its standard CW structure, and $(C., \partial)$ the chain complex that computes its *cellular* homology groups. Compute the associated cohomology groups $H^n(\Sigma; G)$ with $G = \mathbb{Z}$ and $\mathbb{Z}_2$. (Hint : you need, of course, to use the classification theorem of compact surfaces and make a distinction between the orientable and non-orientable case, for every genus g).

Exercise 3.

Compute the cohomology groups of the (standard) cellular chain complex of $\mathbb{R}\mathrm{P}^n$ with coefficients $G = \mathbb{Z}$ and $G = \mathbb{Z}_2$, for every $n \geq 1$. Verify your answer by using the Universal Coefficient Theorem for cohomology.